SRI AKILANDESWARI WOMEN'S COLLEGE, WANDIWASH

FUZZY SET THEORY

Class : I PG MATHEMATICS

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Brief History

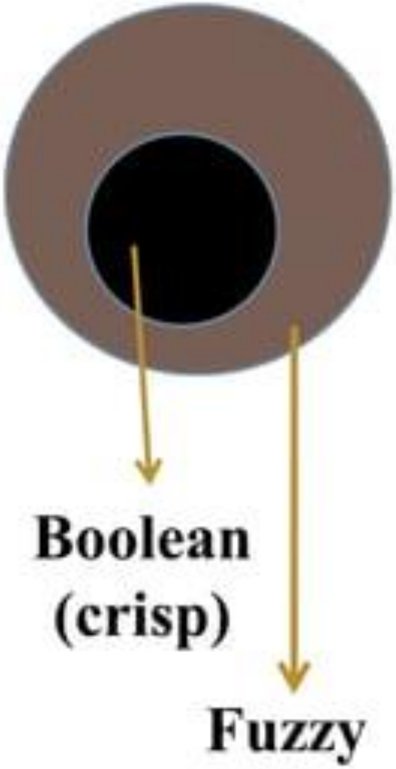
- Classical logic of Aristotle: Law of Bivalence "Every proposition is either True or False(no middle)"

- Jan Lukasiewicz proposed three-valued logic : True, False and Possible

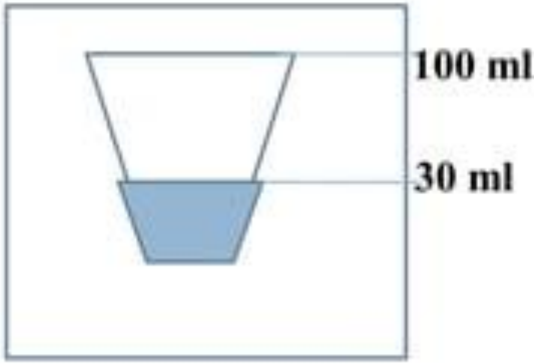
- Finally Lofti Zadeh published his paper on fuzzy logic-a part of set theory that operated over the range [0.0-1.0]

What is Fuzzy Logic?

- Fuzzy logic is a superset of Boolean (conventional) logic that handles the concept of partial truth, which is truth values between "completely true" and "completely false".
- Fuzzy logic is **multivalued.** It deals with **degrees of membership** and **degrees of truth.**
- Fuzzy logic uses the continuum of logical values between 0 (completely false) and 1 (completely true).



For example, let a 100 ml glass contain 30 ml of water. Then we may consider two concepts: Empty and Full.



In boolean logic there are two options for answer i.e. either the glass is half full or glass is half empty.

In fuzzy concept one might define the glass as being 0.7 empty and 0.3 full.

Crisp Set and Fuzzy Set

- Classical set theory enumerates all element using

$A=\{a_1,a_2,a_3,a_4\ldots,a_n\}$

Set A can be represented by Characteristic function

$\mu_a(x)=\{$ 1 if element x belongs to the set A

0 otherwise

}

Example: Consider space X consisting of natural number<=12

Prime={x contained in X | x is prime number={2,3,5,7,11}

- In fuzzy set theory elements have varying degrees of membership

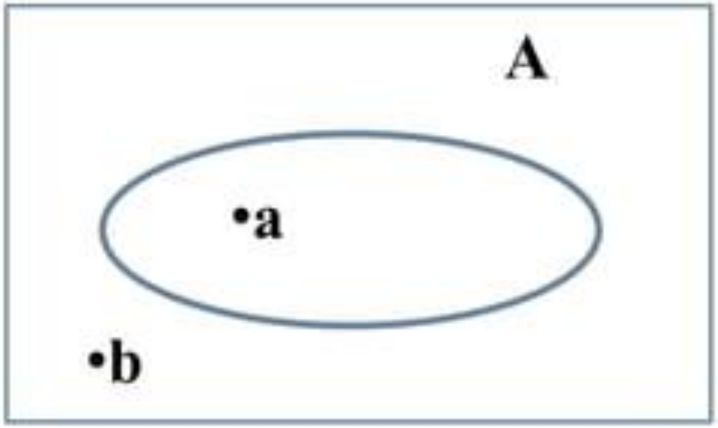
A fuzzy set can be represented by:

A={{ x, A(x) }}

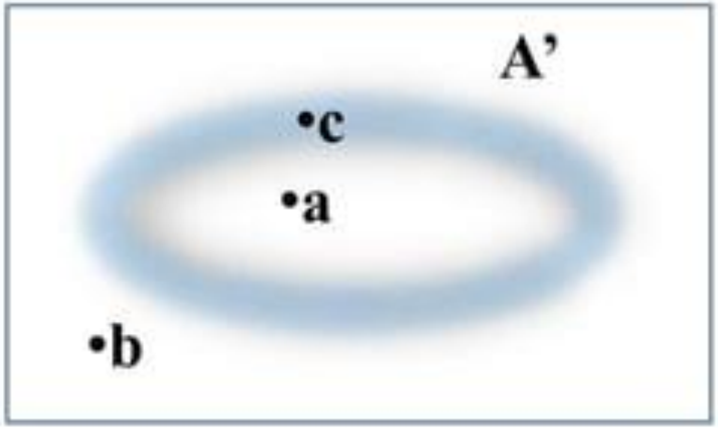
where, A(x) is the membership grade of a element x in fuzzy set

SMALL={{1,1},{2,1},{3,0.9},{4,0.6},{5,0.4},{6,0.3},{7,0.2},{8,0.1},
{9,0},{10,0},{11,0},{12,0}}

Fuzzy Vs. Crisp Set



Crisp set



Fuzzy set

- a: member of crisp set A
- b: not a member of set A

- a: full member of fuzzy set A’
- b: not a member of set A’
- c:partial member of set A’

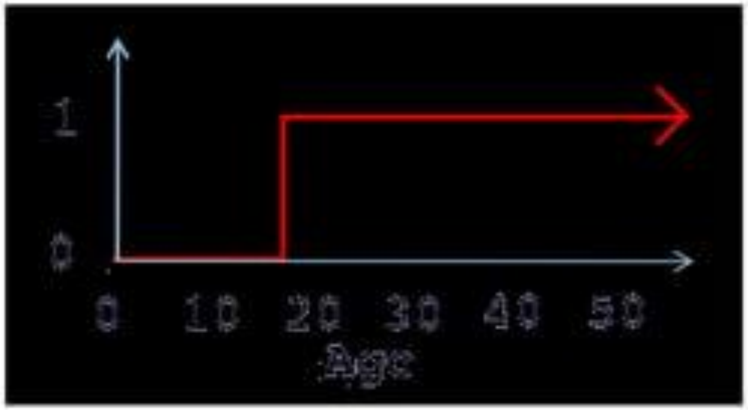
Fuzzy Vs. Crisp Set

Crisp set

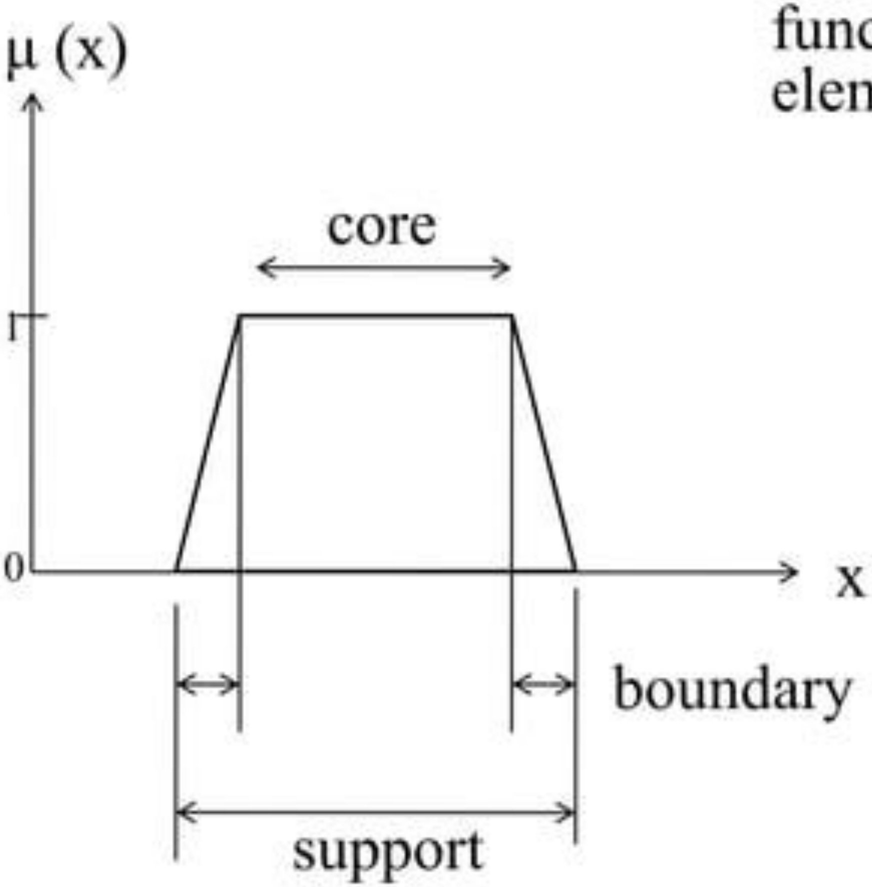
Name	Age	Degree of membership
Sally	5	0
Jenny	18	0
Christen	25	1

Fuzzy set

Name	Age	Degree of membership
Sally	5	0
Jenny	18	0.75
Christen	25	1



Features of a membership function



A membership function is a mathematical function which defines the degree of an element's membership in a fuzzy set.

- Core: region characterized by full membership in set A' i.e. $\mu(x)=1$.
- Support: region characterized by nonzero membership in set A' i.e. $\mu(x) > 0$.
- Boundary: region characterized by partial membership in set A' i.e. $0 < \mu(x) < 1$

Membership Functions

adult(x)= { 0, if age(x) < 16years

(age(x)-16years)/4, if 16years < = age(x)< = 20years,

1, if age(x) > 20years

}

Fuzzy Logic Vs Probability

- Both operate over the same numeric range and at first glance both have similar values:0.0 representing false(or non-membership) and 1.0 representing true.

- In terms of probability, the natural language statement would be "there is an 80% chance that Jane is old."

- While the fuzzy terminology corresponds to “Jane’s degree of membership within the set of old people is 0.80.’

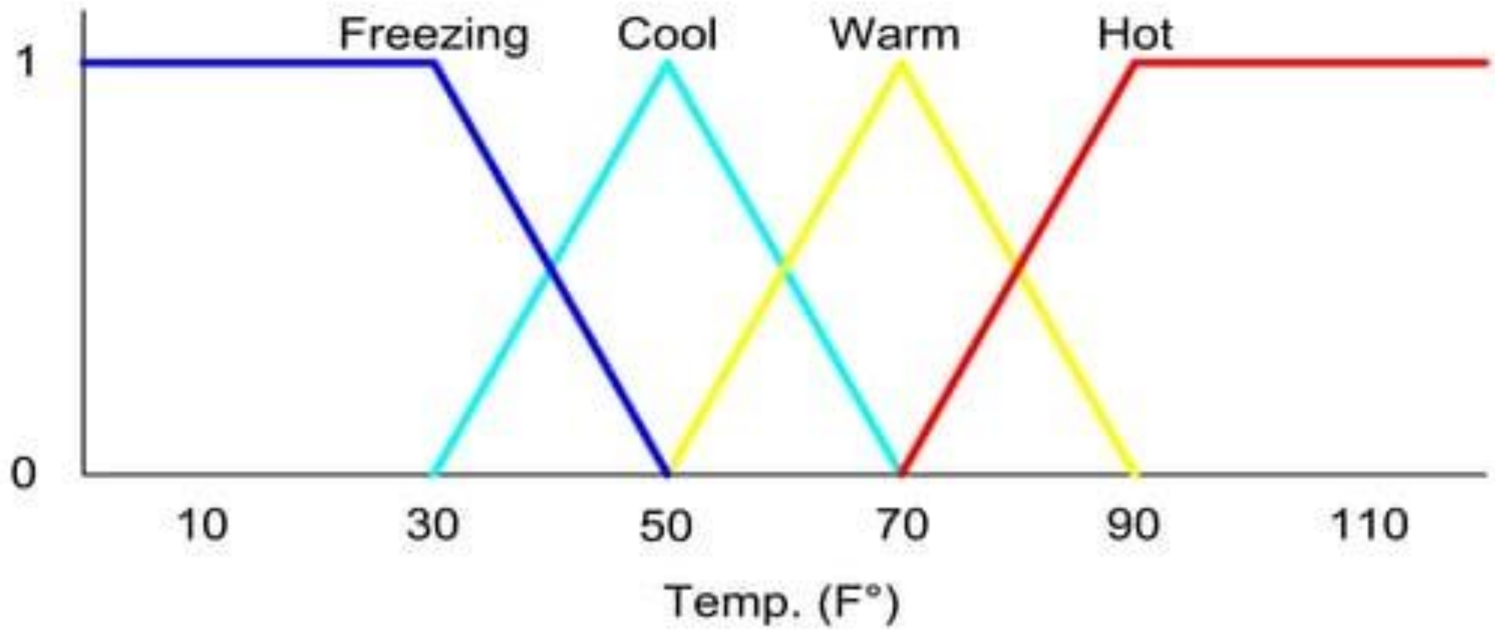
- Fuzzy logic uses truth degrees as a mathematical model of the vagueness phenomenon while probability is a mathematical model of ignorance.

Why use Fuzzy Logic?

- Fuzzy logic is flexible.
- Fuzzy logic is conceptually easy to understand.
- Fuzzy logic is tolerant of imprecise data.
- Fuzzy logic is based on natural language.

Fuzzy Linguistic Variables

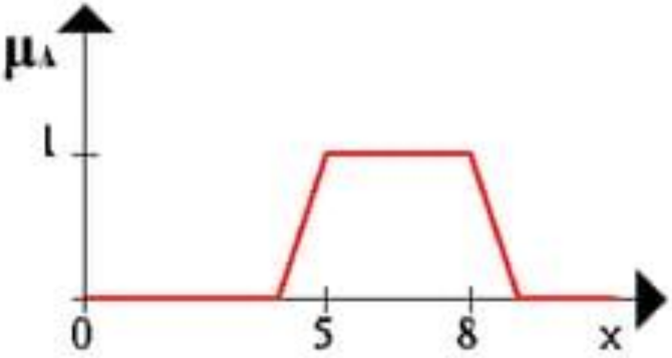
- Fuzzy Linguistic Variables are used to represent qualities spanning a particular spectrum
- Temp: **{Freezing, Cool, Warm, Hot}**



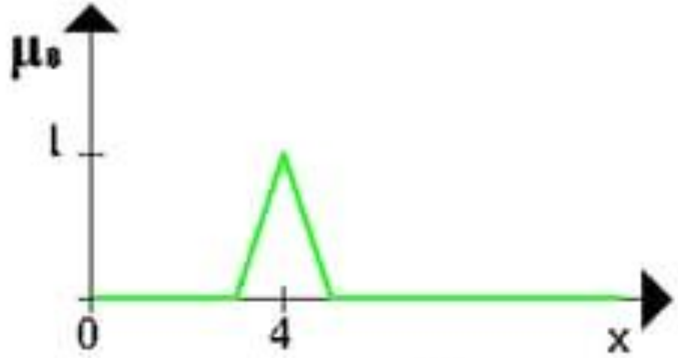
Operations on Fuzzy Set

Consider:

A= {1/2 + .5/3 + .3/4 + .2/5} B= {.5/2 + .7/3 + .2/4 + .4/5}



A



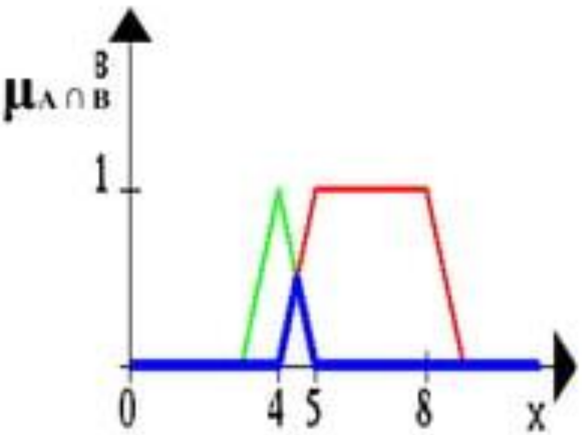
B

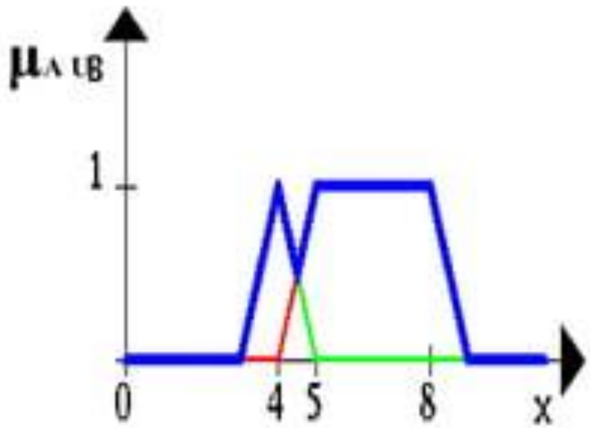
——— >Fuzzy set (A)

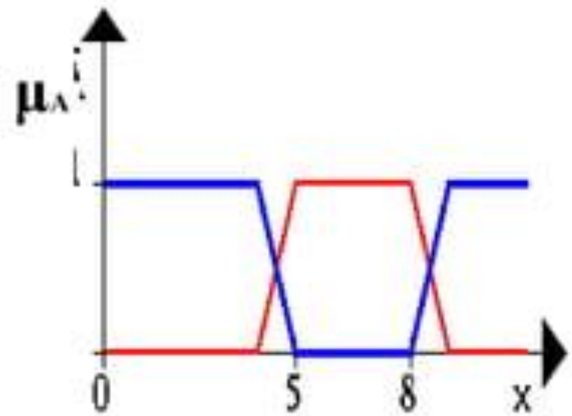
———>Fuzzy set (B)

———>Resulting operation of fuzzy sets

INTERSECTION (A ^ B)	UNION (A ∨ B)	COMPLEMENT (¬A)







$\mu_{A \cap B} = \min(\mu_A(x), \mu_B(x))$

$\{.5/2 + .5/3 + .2/4 + .2/5\}$

$\mu_{A'} = 1 - \mu_A(x)$

$\{1/1 + 0/2 + .5/3 + .7/4 + .8/5\}$

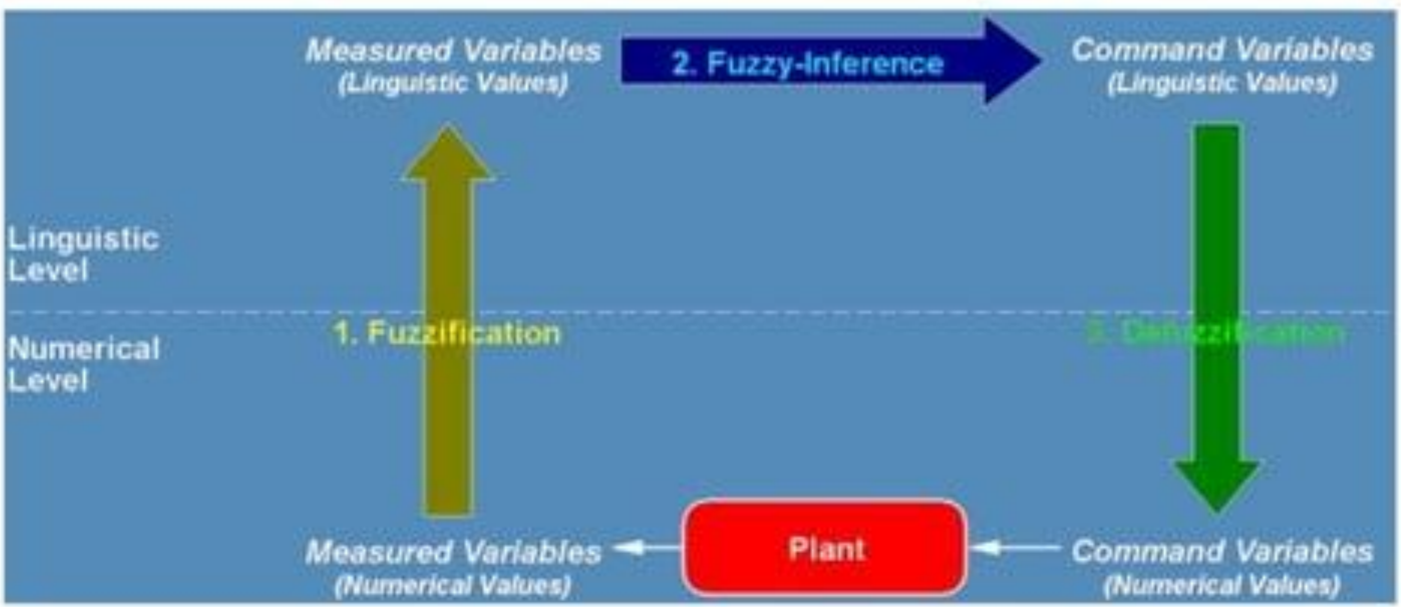
$\mu_{A \cup B} = \max(\mu_A(x), \mu_B(x))$

$\{1/2 + .7/3 + .3/4 + .4/5\}$

Example Speed Calculation

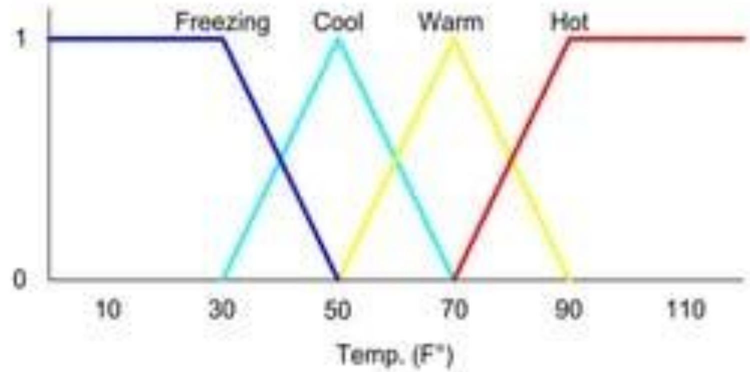
How fast will I go if it is

- 65 F
- 25 % Cloud Cover ?

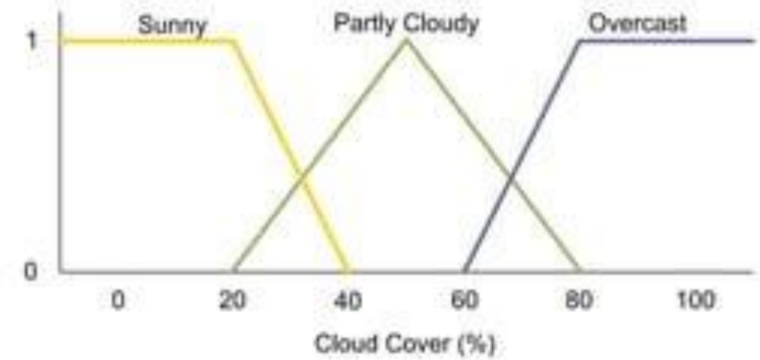


Input:

Temp: {Freezing, Cool, Warm, Hot}

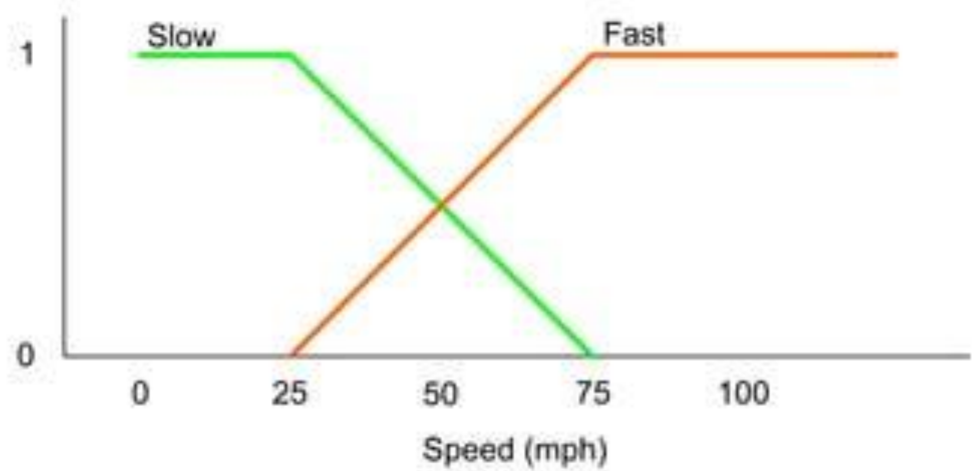


Cover: {Sunny, Partly cloudy, Overcast}



Output:

Speed: {Slow, Fast}



Rules

- If it's Sunny and Warm, drive Fast
  $Sunny(Cover) \wedge Warm(Temp) \Rightarrow Fast(Speed)$

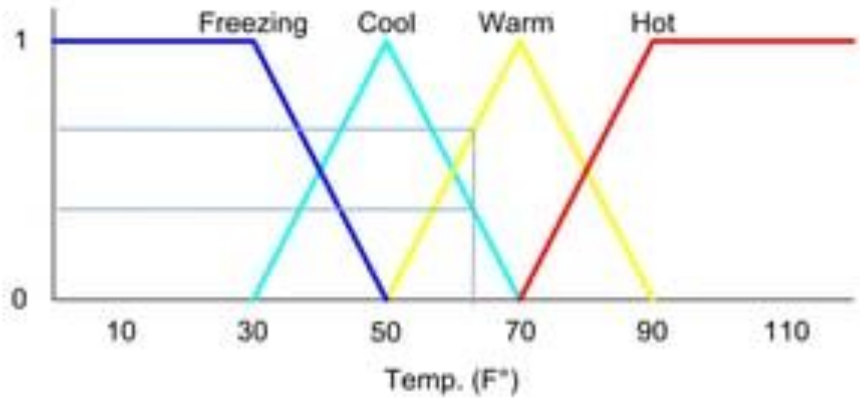
- If it's Cloudy and Cool, drive Slow
  $Cloudy(Cover) \wedge Cool(Temp) \Rightarrow Slow(Speed)$

- Driving Speed is the combination of output of these rules...

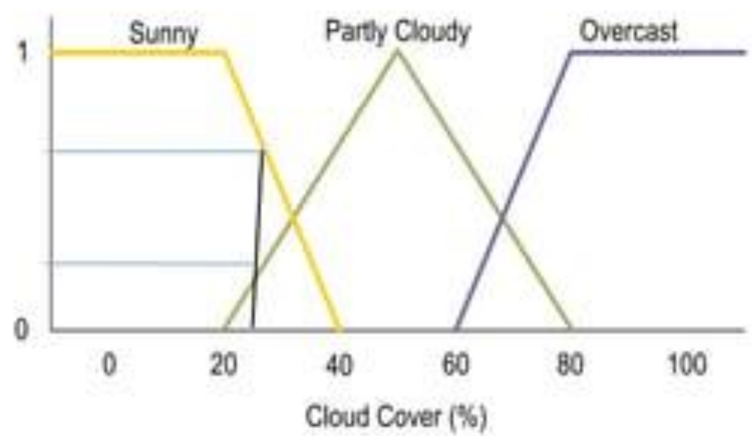
Fuzzification:

Calculate Input Membership Levels

- 65 F ⇒ **Cool = 0.4, Warm= 0.7**



- **25% Cover ⇒Sunny = 0.8, Cloudy = 0.2**



Calculating:

- If it's Sunny and Warm, drive Fast

$$\text{Sunny(Cover)} \wedge \text{Warm(Temp)} \Rightarrow \text{Fast(Speed)}$$

$$0.8 \wedge 0.7 = 0.7$$

$$\Rightarrow \textbf{Fast = 0.7}$$

- If it's Cloudy and Cool, drive Slow

$$\text{Cloudy(Cover)} \wedge \text{Cool(Temp)} \Rightarrow \text{Slow(Speed)}$$

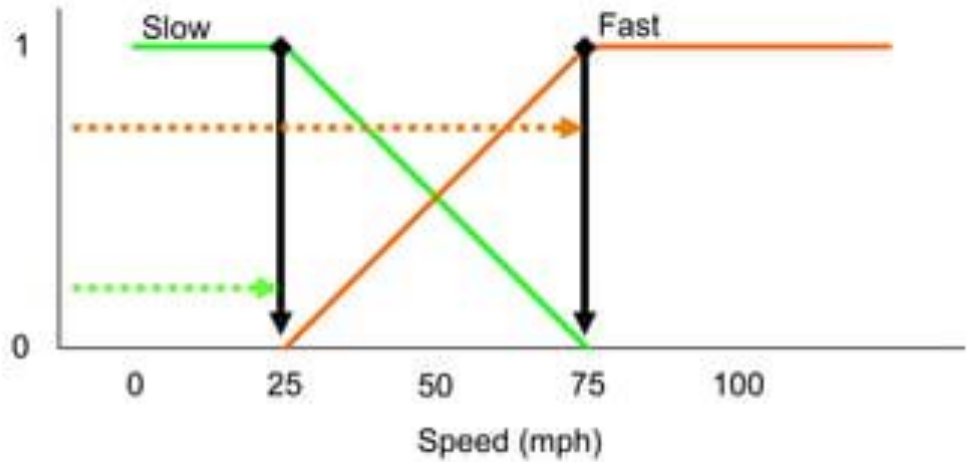
$$0.2 \wedge 0.4 = 0.2$$

$$\Rightarrow \textbf{Slow = 0.2}$$

Defuzzification:

Constructing the Output

- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

- Speed = weighted mean
  = (2*25+7*75)/(9)
  = 63.8 mph

Fuzzy Applications

- **Automobile and other vehicle subsystems :** used to control the speed of vehicles, in Anti Braking System.
- **Temperature controllers :** Air conditioners, Refrigerators
- **Cameras :** for auto-focus
- **Home appliances:** Rice cookers , Dishwashers , Washing machines and others

Drawbacks

- Fuzzy logic is not always accurate. The results are perceived as a guess, so it may not be as widely trusted .
- Requires tuning of membership functions which is difficult to estimate.
- Fuzzy Logic control may not scale well to large or complex problems
- Fuzzy logic can be easily confused with probability theory, and the terms used interchangeably. While they are similar concepts, they do not say the same things.